Computational Geometry Final Exam 2013

Administration

The final should be submitted by 5pm on Friday, December 6, 2013. You can bring it to my office (ITEB 361) and slide it under the door if I am not there. It may be submitted by email if sent by 1pm on that day. Expect to get a confirmation from me by 4pm if you send it by email. Otherwise, send me another email.

You are not permitted to work together or discuss the problems on the final. If you have a question, ask me.

The questions are weighted evenly.

1

Let $p = \begin{bmatrix} u \\ v \end{bmatrix}$ be a point in the plane. Let $p^\star = \{\begin{bmatrix} x \\ y \end{bmatrix} \mid \begin{bmatrix} u \\ v \end{bmatrix}^\top \begin{bmatrix} x \\ y \end{bmatrix} = -1\}$ be a line associated to p. This is a duality between points and lines that is different from the one we saw in class. For a given point, we can find a dual line using the above definition and for a given line ℓ, we can (almost always, see part 2) find a dual point p such that $p^\star = \ell$. The different parts of this question will require you to work out some important features of this duality.

1. Give the point dual to the following lines. Note that you can check your work by checking that the dual of the dual of a line results in the original line.
   - $\{\begin{bmatrix} x \\ y \end{bmatrix} \mid \begin{bmatrix} 3 \\ 1 \end{bmatrix}^\top \begin{bmatrix} x \\ y \end{bmatrix} = -1\}$
   - $\{\begin{bmatrix} x \\ y \end{bmatrix} \mid \begin{bmatrix} x \\ y \end{bmatrix}^\top \begin{bmatrix} 5 \\ 4 \end{bmatrix} = -1\}$
   - $\{\begin{bmatrix} x \\ y \end{bmatrix} \mid \begin{bmatrix} 8 \\ 4 \end{bmatrix}^\top \begin{bmatrix} x \\ y \end{bmatrix} = 4\}$
   - $\{\begin{bmatrix} x \\ y \end{bmatrix} \mid y = mx + b\}$
2. As always, when attempting to define a projective duality on the Euclidean plane, we need to add in a line of "points at infinity". Which lines dualize to a point at inifinity? Note that the fourth part of question 1.1 might give a hint.
3. Prove that incidence is preserved by this duality. That is, show that $p \in \ell$ if and only if $\ell^\star \in p^\star$.
4. In homogeneous coordinates, we represent a point $p = \begin{bmatrix} u \\ v \end{bmatrix} \in \mathbb{R}^2$ as a vector $\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \in \mathbb{R}^3$. Moreover, we represent a line as $\ell = \{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \mid \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}^\top \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0\}$. Show that $p^\star = \ell$ using the duality given above.

5. Let's say that the dual of a set S is the intersection of the duals of the points. That is

$$S^\star = \bigcap_{p \in S} p^\star.$$

Which of the following words best describes a set whose dual is a single point? Circle one.

- connected
- compact
- collinear
- bounded
- Delaunay
- locally Delaunay

2

In class we saw some pseudocode to do nearest neighbor search by adaptive range search in a binary decomposition. The trick was to pass the nearest point found so far as a parameter to the function. Write a new version of this pseudo code to find the second nearest neighbor of a query q. This is a point p such that at most one other point p' is closer to q than p. The signature of this function should look like $2\mathrm{ANN}(T, q, p_1, p_2)$, where p_1 and p_2 are the nearest and second nearest points found thus far. It returns a pair of points (p_1, p_2) of the nearest and second nearest points to q. Only give the main function. You may assume that the tests to check if a tree node intersects a ball have been given to you.

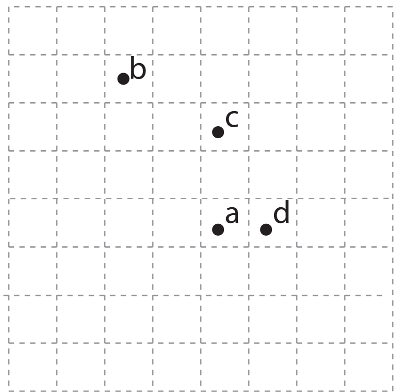
3

- Given 4 points a, b, c, and d in convex position in the plane such that d is in the interior of the circumcircle of $\triangle abc$. What are the triangles of the Delaunay triangulation of $\{a, b, c, d\}$?
- Given 5 points in the plane, is it possible that every pair of points forms an edge of the Delaunay triangulation? Just give a yes or no answer with a one sentence justification.
- Given 6 points in convex position, how many different ways are there to triangulate the points.
- What are necessary and sufficient conditions on a set of points to guarantee that the Delaunay triangulation is unique.

4

Consider the points in the unit square depicted below.

1. Subdivide the square into the minimal quadtree that separates each point into its own square. The dotted lines are there to help you. The full square should be the root.



2. Draw the 4-ary tree T that represents this quadtree. Be sure to order the leaves from left to right according to the z-order. That is, top-left, top-right, bottom-left, and then bottom-right. Don't forget to label the points in the leaves.

3. Draw the compressed version of T.

4. List the points in increasing order of the z-order.

5. Draw the balanced version of T.