How did we get here?

Models of Computation

I. Ruler and Compass
- Euclid
- Lines + Circles
- Intersections from pictures

II. Real RAM
- Descartes
- Coordinates

III Comparison Model
- Linear Predicates (Determinants)
  (Yes/No/Maybe Questions)
- Relation to Sorting → Convex Hulls

IV floating point + exact arithmetic
- Real computers
- "Filtered" predicates

Last-time: Polygons, Jordan Curve Thm

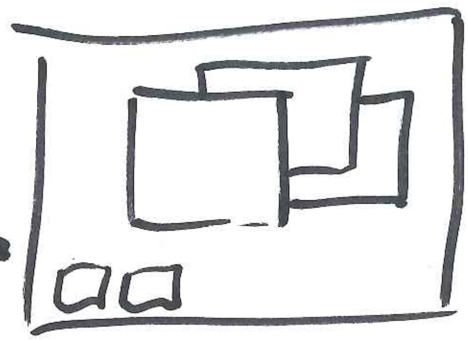
$\Rightarrow$ Geometric Graphs

This was a warmup to help us start thinking about graphs as geometry.

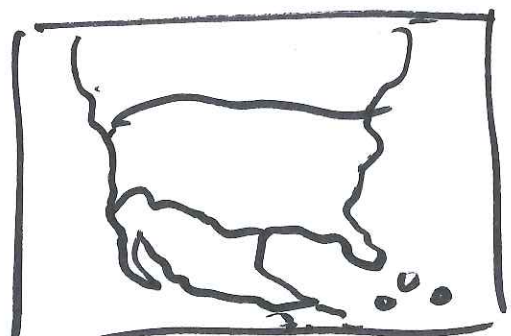
2^{+1} big applications:

* Screens

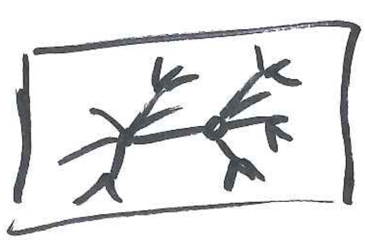
you clicked where?



* Maps



Bonus: * Data



Planar Straight-line Graphs

<u>Def</u> A <u>graph</u> is a pair (V,E) where V is a finite set called vertices and E is a collection of pairs called edges in V (i.e. $E \subseteq \binom{V}{2}$).

<u>Examples of Graphs to Know</u>

Don't say "click"

(1) <u>Clique</u> (Complete Graph) $E = \binom{V}{2}$

(2) <u>Path</u>

(3) <u>Cycle</u>

<u>Def</u> A <u>drawing</u> of $G=(V,E)$ <u>in the plane</u> is a representation of G s.t. vertices are distinct points in $\mathbb{R}^2$ and edges are simple arcs where

(1) arcs start and end at corresponding edge vertices.

(2) No arc touches a vertex other than its end points

(3) arcs only intersect at common end points.

Def A graph G is planar if there exists a ~~g~~drawing of G in the plane.

Def A planar straight-line graph (PSLG) is a drawing of a graph G in the plane where all edges are represented by straight line segments.

Thm [Fáry 1948] Every planar graph can be drawn as a PSLG.

This gives a geometric definition of planarity to complement our topological definition.

Examples
(non-crossing) Polygonal Chain - PSLG of a path
(simple) Polygon - PSLG of a cycle

PSLGs are easier to represent on a computer because we only need G and vertex positions.

Jordan Curves give/define <u>faces</u> in a PSLG.

<u>Def</u> A <u>face</u> in PSLG $\mathcal{D}$ is a connected component of $\mathbb{R}^2 \setminus \mathcal{D}$.

this is $\boxed{\text{not}}$ graph components

The "infinite" or "unbounded" face counts as a face.

Euler's Formula

Let F be the set of faces of a PSLG.

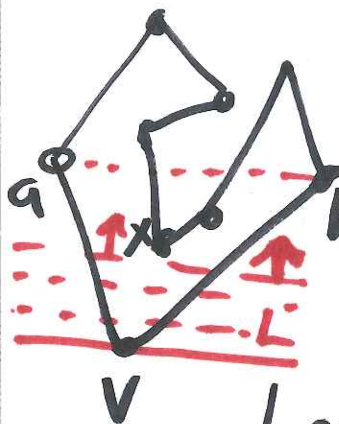
$$|V| - |E| + |F| = 1 + \#\left\{\begin{array}{c}\text{connected} \\ \text{components} \\ \text{of graph}\end{array}\right\}.$$

Every Polygon has a triangulation.

pf Induction on number of vertices.

Base Case: $n=3$. It's a triangle. $\Rightarrow$ done.

If $n>3$, it will suffice to find one diagonal (a straight line segment between nonadjacent vertices thru the interior.)



Let v be the lowest vertex.
Let a, b be its neighbors (If $\overline{ab}$ is a diagonal, we're done)
Let L be line parallel to ab thru v.
Sweep L upwards until it touches a vertex x inside the triangle Δabv.
Observe that vx must be a diagonal (by convexity).