CSE3502 - Theory of Computation January 27, 2014

Due: Before class on January 28, 2014

Homework 1

1. Did you do the reading? YES/NO/SORTA
2. Did you do the reading before class? YES/NO/SORTA
3. How long did you spend on this homework (rounding up)? ___________hours.

1 Sets

Objective: Read and write formal descriptions of sets. Manipulate sets using basic set operations.
Give a formal description of the following sets.

- The set of prime numbers less than 15.
- The set consisting of the strings *aba* and *baa*.
- The set of integers less than 8.
- The set containing the empty string.
- The set that doesn't contain anything at all.

Let $X = \{1, 2, 3, 4\}$ and let $Y = \{2, 4\}$.

- Is X a subset of Y?
- Is Y a subset of X?
- What is $X \cap Y$?
- What is $X \cup Y$?
- What is $Y \times X$?
- What is $X \setminus Y$?
- What is the power set of Y?

2 Proofs

Objective: Write clear, correct proofs using different techniques. Identify flawed arguments.

The questions in this section all deal with something called *graph isomorphism*. Let $G = (V, E)$ and $H = (U, D)$ be two undirected graphs. An *isomorphism* is a bijection $f : V \to U$ such that $\{u, v\} \in E$ if and only if $\{f(u), f(v)\} \in D$ (Recall that a bijection is a one-to-one, onto function).

If there exists an isomorphism between the vertex sets of graphs G and H, we say that G and H are *isomorphic* and denote this $G \cong H$.

- Prove that $\cong$ is an equivalence relation. Recall, this means that $\cong$ is symmetric, reflexive, and transitive.

- Find the bug in the following inductive proof that in any set of h graphs, every pair is isomorphic. Express your answer in one clear, concise sentence.

  Base case: If $h = 1$. In any set containing just one graph, the one graph is isomorphic to itself since $\cong$ is reflexive.

  Inductive Step: For $k \geq 1$, assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set X of $k + 1$ graphs. We show that every pair of graphs in X is isomorphic. It will suffice to show that any two graphs G and H in X are isomorphic. By induction, H is isomorphic to every graph in the set $X_1 = X \setminus G$ because $|X_1| = k$. Similarly, by induction, G is isomorphic to every graph in the set $X_2 = X \setminus H$. Let K be a graph in $X_1 \cap X_2$. So, $G \cong K$ and $H \cong K$. Thus, $G \cong H$ because $\cong$ is transitive.

- The *complement* of a graph $G = (V, E)$ is another graph $G^\circ = (V, E^\circ)$ with the same vertex set V and contains an edge for every $\{u, v\}$ such that $\{u, v\} \notin E$. Let G be the cycle graph of length 5 with the vertex set $\{0, 1, 2, 3, 4\}$. The edges in G are the pairs $\{\{i, j\} \mid j \equiv i + 1 \pmod 5\}$. Prove that $G \cong G^\circ$ by giving the isomorphism.