CSE3502 - Theory of Computation April 9, 2014

Due: Before class on April 15, 2014

Homework 7

1. Did you do the reading? YES/NO/SORTA
2. Did you do the reading before class? YES/NO/SORTA
3. How long did you spend on this homework (rounding up)? ___________hours.

1 Time Complexity Classes

1.1 The book asserts that every language that can be decided in time $o(n \log n)$ time is regular. Use this fact to prove that $\text{TIME}(n \log(\log n)) \setminus \text{TIME}(n) = \emptyset$.

2 The Ozgur Machine does not exist!

We saw in class (*and its also in the book*) that the acceptance problem for Turing machines is undecidable. That is, the language

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } w \in L(M)\}$$

is undecidable. We will use this theorem to show that there cannot exist a Turing machine that can reliably decide whether or not you submitted correct answers to some of the problems from homework 5. Recall that the language A was defined as

$$A = \{(0 \cup 1)^a (1 \cup 2)^b (2 \cup 3)^c \mid a \geq b\}.$$

In homework 5, you were asked to give a Turing machine to recognize A. Consider the language of correct answers:

$$Z = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = A\}.$$

We will show that Z is undecidable. At least one answer T_A was given in the solutions that decides A.

Suppose for contradiction that there exists a Turing machine OZGUR that can decide Z. Then, we will create a new Turing machine T that works as follows.

$T =$ On input $\langle M, w\rangle$

1. Write the description of a TM R that works as follows

   $R =$ On input x

   Run T_A on x.

   If T_A accepts x then accept.

   If T_A rejects x then run M on input w and accept if M does

2. Run OZGUR on input $\langle R\rangle$, accept if OZGUR rejects and reject if OZGUR accepts.

Note that the machine R has the machine M and the string w "hard-coded" into it. These are not part of the input to R.

2.1 Is R necessarily a decider? Why or why not? Recall that a decider always terminates.

2.2 Is T a decider? Why or why not?

2.3 What is $L(R)$ in terms of w, A, Σ, and $L(M)$?

2.4 Prove that T decides A_{TM}.

Finally, after proving in 2.4 that T decides A_{TM}, we can conclude that OZGUR cannot exist, because there cannot be a TM that decides A_{TM}.

3 Big-O and little-o

Let f and g be functions such that $f(n) = O(n^2)$ and $g(n) = O(n)$.

3.1 Let $h_1(n) = f(n) + g(n)$. Is it necessarily true that $h_1(n) = O(n^2)$? Give a proof.

3.2 Let $h_2(n) = f(n)g(n)$. Is it necessarily true that $h_2(n) = o(n^3 \log n)$? Give a proof.

3.3 Let $h_3(n) = \frac{f(n)}{g(n)}$. Is it necessarily true that $h_3(n) = O(n)$? Give a proof.

3.4 Let $h_4(n) = 2^{O(\log(\log n))}$. Is it necessarily true that $h_4(n) = O(\log n)$? Give a proof.